

Kugel

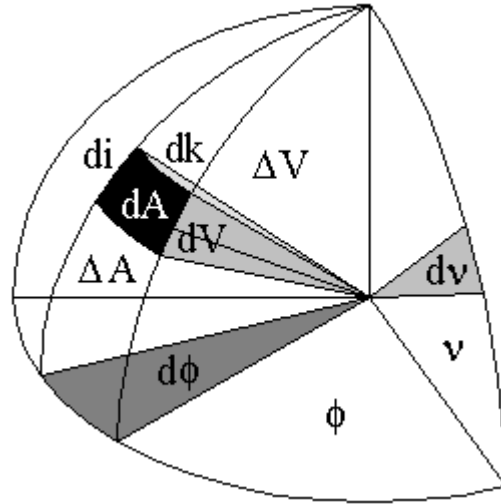


Abbildung 1: Kugelsegment - Längen und Winkel

$$\begin{aligned}
 a &= R \cos \nu \\
 dk &= a \tan d\phi = a d\phi \\
 di &= R \tan d\nu = R d\nu \\
 dk &= R \cos \nu d\phi
 \end{aligned}$$

Oberfläche

$$\begin{aligned}
 dA &= dk di \\
 dA &= R \cos \nu d\phi R d\nu = R^2 \cos \nu d\phi d\nu \\
 \Delta A &= R^2 d\phi \int_0^{\pi/2} \cos \nu d\nu \\
 \Delta A &= R^2 d\phi \sin \nu \Big|_0^{\pi/2} \quad (\sin \pi/2 = 1) \\
 A &= \int_0^{2\pi} R^2 d\phi \\
 A &= R^2 \int_0^{2\pi} d\phi = R^2 2\pi \quad (\text{halbe Kugel}) \\
 A_{\text{ganz}} &= 4\pi R^2
 \end{aligned}$$

Volumen

$$\begin{aligned}
 dV &= \cos \nu d\phi d\nu \int_0^R r^2 dr \\
 dV &= \cos \nu d\phi d\nu \frac{R^3}{3} \\
 \Delta V &= d\phi \frac{R^3}{3} \int_0^{\pi/2} \cos \nu d\nu \\
 \Delta V &= \frac{R^3}{3} d\phi \sin \nu \Big|_0^{\pi/2} \quad (\sin \pi/2 = 1) \\
 V &= \frac{R^3}{3} \int_0^{2\pi} d\phi \\
 V &= \frac{R^3}{3} 2\pi \quad (\text{halbe Kugel}) \\
 V_{\text{ganz}} &= 4\pi \frac{R^3}{3}
 \end{aligned}$$